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Commutativity of Rings with Constraints on Pair of Automorphisms

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Abstract

The aim of this work is to establish some sufficient conditions for the commutativity of rings with unity (For details, see Theorem 2.1), conditions which are somewhat close to the Conjecture (C).

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1. Introduction

Throughout, R will denote an associative ring with unity 1 and $Z(R)$ its center. As a consequence of Jacobson's theory of algebras,Jacobson [7] proved that an algebraic algebra without nilpotents over a finite field is commutative. As a corollary, Jacobson deduced that in a ring R there exist an integer $n > 1$ such that for every $x \in R$, $x^n = x$, then R is commutative. This result implies , in particular, the celebrated theorem of Wedderburn that any finite division ring is a field. There have been in recent years several
generalizations and variations of Jacobson's result but proofs of all these commutativity theorems use "transcendental" methods in the sense that they use Zorn's lemma implicitly
or explicitly (For details, see References [1] & [6]).
In 1951, Herstein [5] proved that if, for each $x \in R$, there exists an integer $n(x) > 1$ such that $x^{n(x)} - x \in Z(R)$, then R is commutative. This result was shown by essential use of Zorn's lemma. Also above result has been extended by Kaplansky [8] as follows: Let R be a division ring and there exists a fixed non-zero polynomial $f(X) \in Z(R)[X]$ such that $f(t) \in Z(R)$ for every $t \in R$. Then R is commutative.