JKAU: Eng. Sci., **vol. 14 no. 1**, pp. 65-76 (1423 A.H./ 2002 A.D.)

An Adaptive Variable Structure Controller for Linearizable Systems

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ABSTRACT. In this work, we designed a Variable Structure Controller (VSC) for uncertain linearizable systems. The concept of augmented plant together with the identification of the uncertainty through an observer has been used to reduce the chattering. A simulation study is reported in illustration.

KEY WORDS. Feedback Linearization, Variable Structure Control, parameter uncertainties.

1. Introduction

Recent developments in the theory of geometric nonlinear control provide powerful methods for control design of Nonlinear systems. The feedback linearization scheme is a powerful method as it is based on exact cancellation of the nonlinearities present in the system. The feedback linearization approach requires a perfect model of the system in order to achieve linearization of the closed loop system.

However, for many real systems, there often exist inevitable uncertainites in their constructed models. In addition, there exists uncertainty in the parameters that are not exactly known or are difficult to estimate. Therefore, the design of a robust controller that deals with un- certainties in the parameters of a Nonlinear system is very important.

Variable Structure Control (VSC) with sliding mode is capable of making a control system very robust with respect to system parameter variations and external disturbances.

The theory of Variable Structure Systems was first proposed and studied in the early 1950s by Soviet researchers [1,2]. VSC is capable of making a control system very robust with respect to system parametric uncertainties and external disturbances.

But the main concern when using the VSC technique is to avoid the excessive chatter. In the steady-state, chattering appears as a sustained oscillation around the desired equilibrium point and may excite unmodelled high-frequency dynamics of the system which may jeopardize system stability. Therefore, it is desirable to seek effective methods of suppressing the chatter. Espana et al. [3] did not take parameter variations and external disturbances into consideration in the design of the VSC. Chang et al. [4] presented a new adaptive chattering alleviation algorithm, but their work doesn't deal with the effect of external disturbances. Zaremba et al. [5] used the idea of hierarchical VSC with on-line iterative tuning of sliding hyperplanes and control function switching terms. The algorithm is based on defining certain local performance indices instead of a global one and is shown to improve the response of the system considerably. AL-Sunni et al.[6] proposed a new Hybrid scheme employing both adaptive and fuzzy logic schemes to smooth out the chattering phenomenon and simultaneously increase the speed of the system response. One way of reducing the steady-state error for systems represented in state space is by inserting an integrator in the feedforward path between the error comparator and the plant. This is known as augmented error approach. Billing et al. [7] proposed the insertion of a low pass filter ahead of the plant to smooth out the VSC output signal. Nassab [8] proposed a filtering method for the case of linear systems only. In Al-Sunni et al. [9] , an adaptive VSC for linearizable systems is proposed using the measurement of the states.

Marino et al. [10] presented a state feedback control which achieves asymptotic tracking of signals generated by a known exosystem for SISO nonlinear systems with unknown parameters entering linearly. Here, it is assumed that the tracking dynamics are not affected by unknown parameters. Kosmatopoulos et al. [11] proposed a switching adaptive controller that overcomes the problem of computation of the feedback control law when the identification of the model becomes uncontrollable. The proposed controller requires knowledge of the sign and lower bound of the input vector field. We propose a new methodology to design a robust controller for linearizable systems. By using the concept of Feedback Linearization, the Nonlinear System is firstly linearized and then a Variable Structure Controller is designed and the chatter in the control signal is reduced using the Plant Augmentation Methodology.

The organization of this paper is as follows. Sec.2 gives a background for the adaptive VSC design for linear systems. Sec.3 describes in detail the proposed scheme. Sec.4 illustrates the simulation results. Sec.5 concludes the paper.

2. Adaptive VSC Design for Linear Systems

The VSC technique is applied to the augmented plant which consists of the original plant and the pure integrator [8] as shown in Figure 1.

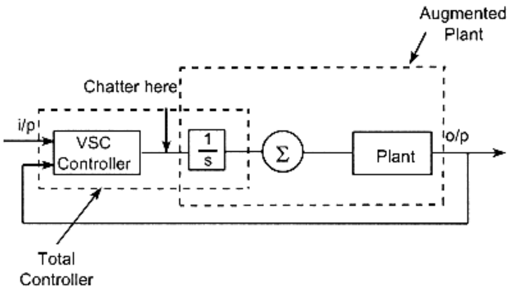

Figure 1 : The augmented plant concept

The system is augmented by a pure integrator, which serves as a buffer, so that the chattering appears at the augmented system rather than the original system. By transforming the augmented system model into a controllable canonical form, the coefficients of the switching surface will define the characteristic equation of the sliding mode. The VSC technique is then applied to the Augmented plant which consists of the original plant and the pure integrator. This procedure for designing a VSC ensures the stability of the augmented system and reduces the control chattering to a great extent.

Assume that the system parameter uncertainties are appearing in the last row of the linear system which is represented in controllable form i.e.

$$\bar{a}_i = a_i + \Delta a_i \tag{1}$$

for i = 1,2 ... n. Then augmenting the system with an integrator and transforming the system to the controllable form, we get

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \\ \dot{x}_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & -a_1 & \cdots & -a_{n-1} & -a_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b \end{bmatrix} u_a + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ -a_n \end{bmatrix} \Delta p \qquad (2)$$

where

$$\Delta p = -\sum_{i=1}^{n} \Delta a_i \; x_i \qquad (3)$$

An (n + 1)th order Luenberger Observer can be used to estimate these uncertainties and disturbances through the observer reconstruction error equations. Since the Luenberger observer is dependent on the plant model, system parametric uncertainties can be estimated by the observer reconstruction error defined as

$$e_i = x_i - \hat{x}_i \qquad i = 1,2 \,....,n \qquad (4)$$

The corresponding Luenberger Observer State equations are represented by

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \vdots \\ \dot{\hat{x}}_n \\ \dot{\hat{x}}_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ -a_1 & -a_2 & \cdots & -a_n & b \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_n \\ \hat{x}_{n+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u_a + \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \\ g_{n+1} \end{bmatrix} e_1 \qquad (5)$$

From the Observer-error state equations, we obtain

$$\Delta p = g_n e_1 + \sum_{i=1}^{n} a_i e_i + \dot{e}_n - b e_{n+1} \qquad (6)$$

For this system, the VSC can be designed as

$$u_a = -\frac{1}{b}[ks(x) + \sum_{i=1}^{n+1} (c_{i-1} - a_{i-1})\, x_i + q\, s\, g\, n\,(s(x)) + (c_n - a_n)\,\Delta p] \tag{7}$$

where c_i are the coefficients of the switching surface.

To make sure that the reaching condition is always satisfied, the magnitude of the signum term is chosen as

$$q = \alpha\, |F\, \Delta p\,| \tag{8}$$

where α is a positive constant (greater than 1) which is used to make q just large enough to handle and neutralize the effects of system uncertainties and F is given by

$$F = \frac{(c_n - a_n)(1-b)}{b} \tag{9}$$

With the correct identification of Δp, the sgn (.) term which is causing the chatter in the control signal will have a magnitude just enough to neutralize the effect of Δp in our scheme. This will reduce the amount of chatter in the control signal.

3. Proposed Scheme

For Nonlinear systems, various State Transformations are used to put the differential equations of the system in one of several possible canonical forms. Consider the SISO nonlinear system,

$$\begin{aligned} \dot{x} &= f\,(x) + g\,(x)\,u \\ y &= h(x) \end{aligned} \tag{10}$$

The above nonlinear system can be linearized through coordinate transformation and nonlinear feedback law to the linear system

$$\dot{z} = Az + bv + \Delta p \tag{11}$$

where Δp is the uncertainty in the parameters of the nonlinear system and

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_1 \end{bmatrix} \tag{12}$$

As seen in the case of Adaptive Design of Linear Systems, the uncertainties can be represented by Δp. Again this Δp can be estimated using the Luenberger Observer. Now, by properly designing the switching surface and estimating the Δp, we can design the controller for the augmented linearized system. Note that $z_{n+1} = v$ where v is the linearizing control and z is the linearizing state vector. And now, by ensuring the reaching condition is satisfied and adjusting the magnitude(q) of the signum term such that it neutralizes the effects of system uncertainties, we can obtain the linearizing control signal. And then the original control signal for the Nonlinear system(u), which is a function of the linearizing state vector and the linearizing control, can be obtained. The proposed scheme is summarized as shown in Figure 2.

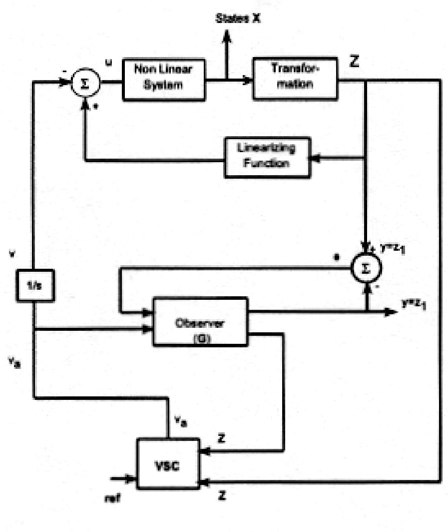

Figure 2: Proposed Scheme

4. Simulation Results

Consider the example [12]

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta x_1^2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= u \end{aligned} \qquad (13)$$

where θ is an unknown constant parameter. This is a "benchmark" example of adaptive nonlinear regulation as it violates the restrictions and assumptions taken into consideration in the uncertainty-constrained schemes and the nonlinearity-constrained schemes proposed in [12].

Restrictions on the type of nonlinearities and assuming some geometric conditions in order to satisfy the matching conditions were set. But this example violates all the geometric conditions and growth assumptions mentioned in [12]. Now let the parameter θ be perturbed by Δθ.

After performing the Feedback Linearization with three coordinate transformations, the final linearized system is given by

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ b\end{bmatrix} v + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}\Delta p \qquad (13)$$

where

$$\begin{aligned}\Delta p = 2\Delta\theta\,[x_1x_3 + 2\theta x_1^2 x_2 + 2\theta^2 x_1^4 + 2\Delta\theta x_1^2 x_2 + 4\Delta\theta x_1^4 \\ + \Delta\theta x_1^3 + x_2 + \theta x_1^2 + \Delta\theta x_1^2\,] + f(t)\end{aligned} \qquad (14)$$

Here $f(t)$ = 20 *sin* (2π0.4t) is the external disturbance.

When the system is augmented by an integrator, the canonical controllable form of the augmented linearized system is given by:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ b \end{bmatrix} v_a + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \Delta p \quad (16)$$

Note that $z_4 = v$. The switching function of the augmented system is

$$S(z) = 290 z_1 + 129\, z_2 + 20\, z_3 + z_4 \quad (17)$$

where the desired eigenvalues are

$$\lambda_{1,2} = -5 \pm j2, \quad \lambda_3 = -10 \quad (18)$$

The augmented control law is given by:

$$v_a = -\left(\frac{1}{30}\right)[10 S(z) + 290 z_2 + 129 z_3 + 20 z_4 + q \operatorname{sgn}(S(z))] \quad (19)$$

Here $b = 20$ and $k = 5$. And q is given by:

$$q = \alpha \left| \frac{(c_3 - a_3)(1-b)}{b} \right| \Delta p \quad (20)$$

where $\alpha = 7$

The Luenberger Observer matrix, G is synthesized by ITAB Butterworth standard forms for closed-loop responses.

G = [52 1360 20800 8000]

Two cases are considered here:

- **Case 1**: When the parameter θ has constant parameter uncertainty with 50% variation. Ref Figure 3.

- **Case 2**: When the parameter θ has constant parameter uncertainty with 100% variation and an external disturbance $f(t) = 20 \sin(2\pi 0.4 t)$ is considered. Ref Figure 4.

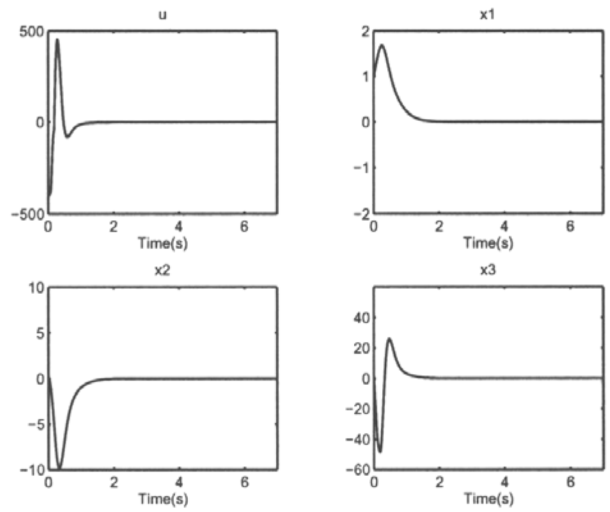

Figure 3: Simulation Results: Case 1.

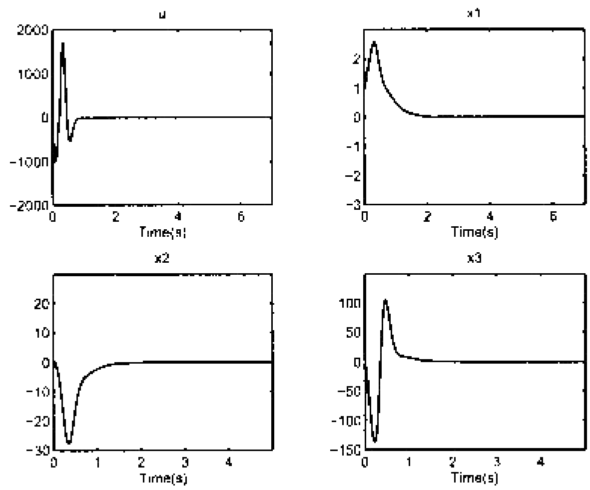

Figure 4: Simulation Results: Case 2.

From the simulation results and the comparison table, we can conclude that our scheme requires less control effort compared to the one presented by Kanellakopulos et al. [12] scheme. The settling time of the states is also improved. In addition, the parameter estimation using our scheme was better compared to [12].

Table 1: Comparison of simulation results with Kanellakopulos et al. Scheme

Measures		Kanellakopulos et al. Scheme		Our Scheme	
		Case1	Case2	Case1	Case2
Control Effort	U_{max}	5857.1	13285	457.37	1710.7
"	U_{power}	2.03×10^6	1.3×10^7	4306.8	4583.8
Settling Time	x_1	4.91	very high	1.44	1.77
"	x_2	5.12	very high	1.78	2.13
"	x_3	5.28	very high	2.09	2.51
Par. Estimate	$\hat{\theta}$	2.4	–	1.97	–

5. Conclusions

A new adaptive VSC scheme has been proposed for the design of a robust controller for uncertain nonlinear system. The main concern when designing a VSC controller (i.e. the chattering phenomenon) is dealt with the augmented plant setup. A "benchmark" example is presented in illustration.

Acknowledgements: The authors would like to thank Systems Engineering Department and the Research Institute of the King Fahd University of Petroleum & Minerals for providing all the facilities needed for this research.

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متحكم متأقلم متغير البنية لأنظمة قابلة لتحويلها لأنظمة خطية

فؤاد السني
قسم هندسة النظم
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معهد البحوث

المستخلص : في هذا البحث ، نصمم متحكم متغير البنيـة لأنظمـة غير خطية وتحمل بعض الأخطاء . ونضيف إلى النظام بعد تحويله إلى نظام خطي وحدة تكاملية ، ونستخدم ملاحـظ للتعـرف علـى الأخطاء من أجل تقليل نسبة القلقة في إشارة التحكم ، نقوم بدراسـة أداء النظام عن طريق المحاكاة .