## Research Article

## **Global Properties of Virus Dynamics Models with Multitarget Cells and Discrete-Time Delays**

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We propose a class of virus dynamics models with multitarget cells and multiple intracellular delays and study their global properties. The first model is a 5-dimensional system of nonlinear delay differential equations (DDEs) that describes the interaction of the virus with two classes of target cells. The second model is a (2n + 1)-dimensional system of nonlinear DDEs that describes the dynamics of the virus, *n* classes of uninfected target cells, and *n* classes of infected target cells. The third model generalizes the second one by assuming that the incidence rate of infection is given by saturation functional response. Two types of discrete time delays are incorporated into these models to describe (i) the latent period between the time the target cell is contacted by the virus particle and the time the virus enters the cell, (ii) the latent period between the time the virus particles. Lyapunov functionals are constructed to establish the global asymptotic stability of the uninfected and infected steady states of these models. We have proven that if the basic reproduction number  $R_0$  is less than unity, then the uninfected steady state is globally asymptotically stable, and if  $R_0 > 1$  (or if the infected steady state exists), then the infected steady state is globally asymptotically stable.

## **1. Introduction**

Nowadays, various types of viruses infect the human body and cause serious and dangerous diseases. Mathematical modeling and model analysis of virus dynamics have attracted the interests of mathematicians during the recent years, due to their importance in understanding the associated characteristics of the virus dynamics and guiding in developing efficient antiviral drug therapies. Several mathematical models have been proposed in the literature to describe the interaction of the virus with the target cells [1]. Some of these models are given