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On fixed point generalizations of Suzuki's method

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ABSTRACT

In order to generalize the well-known Banach contraction theorem, many authors have introduced various types of contraction inequalities. In 2008, Suzuki introduced a new method (Suzuki (2008) [4]) and then his method was extended by some authors (see for example, Dhompongsa and Yingtaweesittikul (2009), Kikkawa and Suzuki (2008) and Mot and Petrusel (2009) [7,10,5,6]). Kikkawa and Suzuki extended the method in (Kikkawa and Suzuki (2008) [5]) and then Mot and Petrusel further generalized it in (Mot and Petrusel (2009) [6]). In this paper, we shall provide a new condition for T which guarantees the existence of its fixed point. Our results generalize some old results.

1. Introduction

Throughout this paper we suppose that (X, d) is a metric space. We denote the family of all non-empty subsets of X by 2^X and the family of all closed subsets of X by $C(X)$. The (generalized) Pompeiu–Hausdorff metric on $C(X)$ is defined by

$$H(A, B) = \max\{\rho(A, B), \rho(B, A)\},$$

where $\rho(A, B) = \sup_{a\in A} D(a, B)$ and $D(a, B) = \inf_{b\in B} d(a, b)$. Note that, $(C(X), H)$ is a complete generalized metric space (in the sense of Luxemburg–Jung; see for example [1]). For a multifunction $F : X \longrightarrow 2^X$, we denote the fixed point set of F by $\mathcal{F}(F)$, that is, $\mathcal{F}(F) = \{x \in X : x \in Fx\}$. In 1969, Kannan proved the following result [2].

Theorem 1.1. *Let (X, d) be a complete metric space and T be a Kannan map on X, that is, for some $\alpha \in [0, \frac{1}{2})$, $d(Tx, Ty) \leq \alpha d(x, Tx) + \alpha d(y, Ty)$. Then T has a unique fixed point.*

Later, Subrahmanyam proved that a metric space X is complete if and only if every Kannan mapping on X has a fixed point [3]. In 2008, Suzuki [4] introduced a new type of mapping and obtained a generalization of the Banach contraction principle in which the completeness can be also characterized by the existence of fixed points of these mappings. Define a nonincreasing function θ form $[0, 1)$ onto $\left(\frac{1}{2}, 1\right]$ by

$$\theta(r) = \begin{cases} 1 & 0 \leq r \leq \dfrac{\sqrt{5}-1}{2} \\ (1-r)r^{-2} & \dfrac{\sqrt{5}-1}{2} \leq r \leq 2^{\frac{-1}{2}} \\ (1+r)^{-1} & 2^{\frac{-1}{2}} \leq r < 1. \end{cases}$$

Suzuki proved the following result in 2008 [4].

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