Hindawi Publishing Corporation
Fixed Point Theory and Applications
Volume 2011, Article ID 812813, 11 pages
doi:10.1155/2011/812813

Research Article

Asymptotically Pseudocontractions, Banach Operator Pairs and Best Simultaneous Approximations

N. Hussain

Department of Mathematics, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

Correspondence should be addressed to N. Hussain, nhusain@kau.edu.sa

Received 3 December 2010; Accepted 12 January 2011

Academic Editor: Mohamed Amine Khamsi

Copyright © 2011 N. Hussain. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The existence of common fixed points is established for the mappings where T is asymptotically f-pseudo-contraction on a nonempty subset of a Banach space. As applications, the invariant best simultaneous approximation and strong convergence results are proved. Presented results are generalizations of very recent fixed point and approximation theorems of Khan and Akbar (2009), Chen and Li (2007), Pathak and Hussain (2008), and several others.

1. Introduction and Preliminaries

We first review needed definitions. Let M be a subset of a normed space $(X, \|\cdot\|)$. The set $P_M(u) = \{x \in M : \|x-u\| = \operatorname{dist}(u, M)\}$ is called the set of best approximants to $u \in X$ out of M, where $\operatorname{dist}(u, M) = \inf\{\|y-u\| : y \in M\}$. Suppose that A and G are bounded subsets of X. Then, we write

$$r_G(A) = \inf_{g \in G} \sup_{a \in A} \|a-g\|,$$
$$\operatorname{cent}_G(A) = \left\{ g_0 \in G : \sup_{a \in A} \|a-g_0\| = r_G(A) \right\}. \tag{1.1}$$

The number $r_G(A)$ is called the *Chebyshev radius* of A w.r.t. G, and an element $y_0 \in \operatorname{cent}_G(A)$ is called a *best simultaneous approximation* of A w.r.t. G. If $A = \{u\}$, then $r_G(A) = \operatorname{dist}(u, G)$ and $\operatorname{cent}_G(A)$ is the set of all best approximations, $P_G(u)$, of u from G. We also refer the reader to Milman [1], and Vijayraju [2] for further details. We denote by $\mathbb{N}$ and $\operatorname{cl}(M)$ $(w\operatorname{cl}(M))$,