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EXISTENCE RESULTS FOR FRACTIONAL DIFFERENTIAL INCLUSIONS WITH SEPARATED BOUNDARY CONDITIONS

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ABSTRACT. In this paper, we apply Bohnenblust-Karlins fixed point theorem to prove the existence of solutions for a class of fractional differential inclusions with separated boundary conditions. Some applications of the main result are also presented.

1. Introduction

Fractional-order models are found to be more adequate than integer-order models in some real world problems as fractional derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes. The mathematical modeling of systems and processes in the fields of physics, chemistry, aerodynamics, electro dynamics of complex medium, polymer rheology, etc. involves derivatives of fractional order. In consequence, the subject of fractional differential equations is gaining much importance and attention. For details and examples, see [1-4, 10, 12, 15-16, 18, 22-24, 26] and the references therein.

Differential inclusions arise in the mathematical modeling of certain problems in economics, optimal control, etc. and are widely studied by many authors, see [7, 20, 25] and the references therein. For some recent development on differential inclusions, we refer the reader to the references [8-9, 13, 19, 21].

Chang and Nieto [7] discussed the existence of solutions for the fractional boundary value problem:

$$\left\{ \begin{array}{l} {}^c_0D^\delta_ty(t)\in F(t,y(t)), \ t\in [0,1], \ \delta\in (1,2),\\ y(0)=\alpha, \ y(1)=\beta, \ \alpha,\beta\neq 0. \end{array} \right.$$

In this paper, we consider the following fractional differential inclusions with separated boundary conditions

(1.1)
$$\begin{cases} {}^{c}D^{q}x(t) \in F(t, x(t)), \ t \in [0, 1], \ 1 < q \le 2, \\ \alpha x(0) + \beta x'(0) = \gamma_{1}, \ \alpha x(1) + \beta x'(1) = \gamma_{2}, \end{cases}$$

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