# King Abdulaziz University
# Faculty of Science
# Department of Statistics

# STAT 210
# THEORY OF PROBABILITY

**COMPUTER SCIENCE STUDENTS**

# CHAPTER II
# Introduction to Probability

# Introduction to Probability

- **Random Experiment** :

  It is an experiment whose possible outcomes are known, but cannot be predicted with certainty.

- **Examples** :

  Example 1. Tossing a coin once

  Example 2. Rolling a die once

  Example 3. Drawing a card from a deck

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- **Sample Space** :

  It is a set whose elements represent all possible outcomes of a random experiment. It is usually denoted by S.

- **Examples** :

In Example (1): S = {Head, Tail} = {H, T}

In Example (2): S = {1, 2, 3, 4, 5, 6}

In Example (3): S = {1, 2, …, 10, Jack, Queen, King}

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- **<u>More Examples</u>** :

**<u>Example</u>** (4): Tossing a coin twice

S = {HH, HT, TH, TT}.

**<u>Example</u>** (5): Rolling a die twice

$S = \{(a, b) \mid a = 1, 2, \ldots 6 \text{ and } b = 1, 2, \ldots 6\}$.

**<u>Example</u>** (6): Tossing a coin 3 times

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

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- **Event** :

  It is a subset of the sample space. It is usually denoted by A, B, … .

**Example** (7). When rolling a die once, list the elements of the events:

A = outcome is an even number = {2, 4, 6}

B = outcome is an odd number = {1, 3, 5}

C = outcome is divisible by 3 = {3, 6}

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**Example** (8): when rolling a die twice, list the elements of the events:

A = getting a sum of 7

= {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

B = getting a sum of at least 9

= {(3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)}

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C = getting a sum of at most 5

= {(1, 1), (1, 2), (2, 1), (1, 3), (2, 2), (3, 1), (1, 4), (2, 3), (3, 2), (4, 1)}

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- **Example** (9): When tossing a coin 3 times

  A = getting one Head = {HTT, THT, TTH}

  B = getting 2 Heads = {HHT, HTH, THH}

  C = getting at least 1 Head

  = {HTT, THT, TTH, HHT, HTH, THH, HHH}

  D = getting at most 2 Head

  = {TTT, HTT, THT, TTH, HHT, HTH, THH}

# Introduction to Probability

## Logical Dictionary

| Symbol | Set Theory | Probability Theory |
|---|---|---|
| A | set | event |
| $A^c$ | Compliment of A | Event “not A” |
| $\phi$ | Empty set | Impossible event |
| S | Universal set | Sample space |
| $A \cup B$ | Union of A and B | Event A or event B |
| $A \cap B$ | Intersection of A and B | Both events A and B |
| $A - B$ | Difference between A and B | Event A, but not B |
| $A \cap B = \phi$ | Disjoint sets | Mutually exclusive events |

# Introduction to Probability

- **<u>Definition of Probability of an event</u>**

**(1)** **<u>Subjective Approach</u>**

It depends on experience and the amount of available information

(2) **<u>Empirical Approach</u>**

It depends on repeating the experiment "n" times and noting the number "m" of occurrence of the event A, and taking

$P(A) = m / n$ for sufficiently large n.

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(3) **Classical Approach**

It depends on assuming that all outcomes are equal y likely. In this case

$$P(A) = \frac{n(A}{n(S)}$$

where

n(A) = number of sample points in A

n(S) = number of sample points in S

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**Mathematical (Axiomatic) Definition**

It depends on Axioms of Probability:

(i) **Axiom 1**: For every event A in S: $P(A) \geq 0$

(ii) **Axiom 2**: $P(S) = 1$

(iii) **Axiom 3**: For mutually exclusive events A and B in S:

$$P(A \cup B) = P(A) + P(B)$$

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## Some Basic Theorems

(1) For any event A in S:

$$0 \le P(A) \le 1$$

(2) For any event A in S:

$$P(A) + P(A^c) = 1$$

(3) For any events A, B in S:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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## Some Basic Theorems

## (continued)

(4) For any events A , B in S:

$$P(A - B) = P(A) - P(A \cap B)$$

(5) For events A, B in such that A subset of B:

$$P(A) \leq P(B)$$

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## Some Basic Theorems

## (continued)

(6) For any events A, B, C in S:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
$$- P(A \cap B) - P(A \cap C) - P(B \cap C)$$
$$+ P(A \cap B \cap C)$$

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**Remark**:

For mutually exclusively events A, B, and C in S:

(1) P(A U B) = P(A) + P(B)

(2) P(A U B U C) = P(A) + P(B) + P(C)

- In general, for mutually exclusively events $A_1$ , $A_2$ , … , $A_n$ defined on S

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i$$

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## <u>Examples Based on Classical Definition</u>

- **<u>Example</u>** (10): When tossing a coin once

  P(Head ) = ½

  P(Tail) = ½

  **<u>Example</u>** (11): When tossing a coin twice

  P( 2 Heads) = ¼, P(2 Tails) = ¼, P(1 Head) = ½

  P(at least 1 Head) = ¾

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- **Example** (12): When tossing a coin 3 times

  P(1 Head) = 3/8,

  P(2 Heads) = 3/8,

  P(3 Heads) = 1/8,

  P( no Heads) = 1/8

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- **Example** (13): When rolling a die once

  P(face 1) = … = P(face 6) = 1/6 =0.167

  P(even number) = 3/6 = 0.5,

  P(odd number) = 3/6 = 0.5,

  P(a number divisible by 3) = 2/6 = 0.333,

  P(a prime number) = 3/6 = 0.5

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- **Example** (14): Rolling two dice once

P(odd number on the first die) = 18/36 = 0.5

P(odd number on second die) = 18/36 = 0.5

P(odd number on both dice) = 9/36 = 0.25

P(getting a sum of 7) = 6/36 = 0.167

P(equal numbers on both dice) = 6/36 = 0.167

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- **Example** (15): Let A and be defined on the same sample space S such that:

$$P(A) = 0.3, P(B) = 0.25, \ P(A \cap B) = 0.07$$

- P(A U B) = P(A) + P(B) – P(A$\cap$ B)

$$= 0.3 + 0.25 - 0.07$$

$$= 0.48$$

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**Example** (16): Let A, B, C be defined on the same sample space S such that

$$P(A) = 0.3, P(B) = 0.25, P(C) = 0.40, P(A \cap B) = 0.07,$$

$$P(A \cap C) = 0.09, P(B \cap C) = 0.08, P(A \cap B \cap C) = 0.03$$

(i) P( at least 1 event will occur) = P(A U B U C)

$= P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C) = 0.74$

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P(only A will occur) = $P(A \cap B^c \cap C^c) = 0.17$

P(only B will occur) = $P(A^c \cap B \cap C^c) = 0.13$

P(only C will occur) = $P(A^c \cap B^c \cap C) = 0.26$

(ii) P(only 1 event will occur)

= P( only A) + P( only B) + P( only C)

= $0.17 + 0.13 + 0.26$

= $0.56$

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P(only A and B will occur) = $P(A \cap B \cap C^c)$ = 0.04

P(only A and C will occur) = $P(A \cap B^c \cap C)$ = 0.06

P(only B and C will occur) = $P(A^c \cap B \cap C)$ = 0.05

(iii) P(only 2 events will occur)

$= P(A \cap B \cap C^c) + P(A \cap B^c \cap C) + P(A^c \cap B \cap C)$

$= 0.15$

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(iii) P(at least 2 events will occur)

$= P(A \cap B \cap C^c) + P(A \cap B^c \cap C) + P(A^c \cap B \cap C) + P(A \cap B \cap C) = 0.18$

(iv) P(no event will occur) $= P[(A \cup B \cup C)^c]$

$= 1 - P(A \cup B \cup C)$

$= 0.26$

# Introduction to Probability

$$P[A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$$
$$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$

(v) $P[A \cap (B \cup C)] = 0.07 + 0.09 - 0.03 = 0.13$

$$P[A \cup (B \cap C)] = P(A) + P(B \cap C) - P(A \cap B \cap C)$$

(vi) $P[A \cup (B \cap C)] = 0.3 + 0.08 - 0.03 = 0.35$